Joseph A. Biello: Candidate’s Statement

My research interests include fluid dynamics, partial differential equations arising from physical problems, and large dynamical systems in general, with an emphasis on the wave dynamics of the tropical atmosphere. Using techniques such as multiple-scales asymptotics, stochastic processes, small scale numerical computation and direct numerical simulations, my work is aimed at isolating the essential processes underlying natural phenomena. In particular, I attempt to use careful analysis of physical models in order to fill in the gaps which arise in large numerical simulations or to scale up the results of smaller numerical models. Recent and ongoing projects can be grouped into three broad categories which I outline below. The review period begins September 2015.

1) Multiscale Asymptotics for the Equatorial Troposphere

The lack of understanding of the effect of the tropics on the Earth’s climate is a major impediment to increasing predictability in the midlatitudes, the latitude band which extends from the tropics to about 60°. I have been working on problems of the organization of nonlinear waves in the tropics and the effect of these waves on midlatitude waves.

Equatorial Kelvin waves driven by midlatitude Rossby waves

In collaboration with G. Kiladis, an observational atmospheric scientist from NOAA, and A. Back (who completed her degree under my supervision in 2016) we are working to understand the generation of tropical Kelvin by extra-tropical Rossby waves. We use both the classical linear theory and the non-linear asymptotic models that I have developed in order to sort out the main effects which establish wave interaction.

An example the atmospheric data shows a recurring pattern where a Rossby wave from the southern hemisphere arrives over the Australian jet, then excites a Kelvin wave over Indonesia; the Kelvin wave travels rapidly eastward bringing precipitation with it. The issue is that, in the classical theory the jet acts as a reflecting latitude allowing no waves to penetrate, so this interaction remains unexplained.

Our work (manuscript in preparation) shows that, whether or not the jet is unstable the presence of strong wind shear near the equator modifies the classical eigenfunctions of the tropics (the Matsuno waves) to such a large extent that the tropical Rossby waves appear significantly like the classical equatorial Kelvin wave. The explanation of this mode requires digging deeper into governing PDEs than has previously been done - and which we as applied mathematicians are more apt to do.

The Kelvin Wave grinder

In studying the nature of the equatorial Kelvin/Rossby wave interaction, I was drawn to the question, “why does the Kelvin wave exist and why is it so different than the other modes of rotating fluids”. Kelvin waves are non-dispersive boundary trapped (or in the case of the tropics, equatorially trapped) gravity waves. They travel only along the boundary and only to the east (i.e. the direction of rotation of the Earth), whereas, their inertia-gravity wave counterparts propagate in either direction.

Along with my graduate student, A Back, we have arrived at a significant result (paper 31). Kelvin waves in the tropics are stable to a wide range of zonal (east/west) shear and observations of Kelvin waves, notwithstanding the coupling I described above, look like the classical linear theory
- with one glaring difference. The observational record shows that there are no Kelvin waves on the largest horizontal scales in the equatorial troposphere. (These are the scales on which the Madden-Julian Oscillation is active, and I will say more about them below). Using the multi-scale asymptotics from my work of 2004 with Majda we found that, since the meridional (north/south) circulation, albeit weaker than the zonal circulation, has a strong convergence at the equator, it is strongly felt by the lowest frequency waves in the tropics. In fact, we have identified a "grinder" mechanism whereby the meridional circulation grinds (thereby dispersing) any low frequency waves - the longest scale Kelvin waves are the most prominent of these.

This is the first identification of a mechanism for this gap in the observational spectrum and we speculate that it may have a significant effect on the onset of the Madden-Julian Oscillation (see below).

I have also begun a collaboration with J. Guerra, a postdoc in the Atmospheric Sciences group at UC Davis where we are studying the predictions of the Kelvin Wave Grinder in a controlled numerical experiment.

**Why Kelvin waves?**

The structure of the equatorial shallow water equations leaves a nagging question; why are there three boundary trapped waves, a Kelvin wave and two mixed Rossby-Gravity waves. Along with T. Dimofte, we were invited by Science Magazine to write a perspectives article on another paper which provided a topological explanation for this question (paper 30). The authors claimed to identify the Kelvin and mixed Rossby waves using a computation of the Berry Phase associated with the shallow water equations. Our article contextualizes that paper both for atmospheric scientists and for mathematicians. Since we wrote the article, we have uncovered subtle errors in the counting arguments used to rationalize these 3 boundary trapped modes. We intend to continue our work to describe the topological origin of the boundary trapped waves and speculate that the Kelvin wave is, in fact, not properly described by the differences in the Berry Phases of gravity and Rossby waves.

With J. Hunter we have been exploring the behavior of Kelvin waves in a circular tank - using the curvature as a small parameter with which to break the symmetry of the flat interface Kelvin wave problem.

**The Madden-Julian Oscillation (MJO)**

My ongoing collaboration with A.J. Majda (NYU) has included M. Remmel (UC Davis) and now, Xiaoyun Niu (grad student ASGG) and Jennifer Piaseczny (grad student ASGG).

Our (with Majda) two most significant paper on this topic have each been cited almost a hundred times, indicating that our multi scale model of the MJO has made a significant impact in tropical meteorology. In our pioneering work, we developed a multiscale asymptotic theory of planetary scale waves in the tropical atmosphere [12,13] which was the first multiscale understanding of the Madden-Julian Oscillation (MJO), a phenomenon described as the "holy grail" of tropical meteorology.

The MJO is a wave of organized convective activity which begins over Indonesia, propagates eastward with a velocity of about 5 ms\(^{-1}\), ends over the tropical eastern Pacific, and has a period of about 40-50 days. It is latitudinally confined to the tropics, but longitudinally global in nature; it is a major component of intraseasonal weather variability in the tropics, and our lack of theoretical understanding of this phenomenon is a major limitation in medium range midlatitude weather
predictability. Though the MJO has been observed in nature, it has yet to be convincingly simulated in large scale computations, the culprit being the lack of resolution of the correct scales of the interaction. Using a multi-scale asymptotic framework, we are able to show the vertical flux of zonal momentum arising from small scales drives to planetary scale organized flows seen in the Madden-Julian Oscillation.

In Biello/Majda (2010) [17], we generalized the multi-scale framework to allow for stronger background flows, which we expect in nature due to the Hadley circulation. X. Niu and J. Piaseczny (my graduate students in the Atmospheric Sciences Graduate Group) are using these multiscale equations with a sub-grid, cloud model to study the development of organization on the planetary scales in the tropics. J. Piaseczny is studying stochastic prescribed forcing, while X. Niu is studying a semi-linear closure which describes the effect of moist convection.

Atmospheric scientists prefer numerical results to analytic results. I have begun a collaboration with F. Rubini (Univ. of Firenze) and P. Ruggieri (Univ. of Bologna) using the European Center for Midrange Weather Forecasting general circulation model of the atmosphere. This is one of the model used for studying anthropogenic global warming, and we are using it in an idealized state in order to test the hypotheses of the multiscale MJO theory.

Matching Tropical and Extra-Tropical Dynamics on Diurnal Timescales

The classical simplification of the equations of fluid dynamics for atmospheric flows on one day to one month time scales are the so-called “quasi-geostrophic equations (QG)”. The development of these equations relies on the balance between the pressure gradient force and the Coriolis force experienced by the circulating atmosphere. This balance fails in the equatorial regions and we know that different dynamics (the Matsuno equations) are appropriate to the tropics. These two simplifications have never been “stitched together” effectively, so that, to this day, there is no consistent simplified system of equations which describe the interaction of midlatitude QG with tropical dynamics.

While working on the ITCZ equations, I discovered a novel way to connect QG to simplified tropical dynamics; it connects not to the Matsuno theory, but to our ITCZ equations. Affecting this matching requires the use of the technique of matched asymptotic expansion, the willingness to discard the assumption that the Matsuno theory was the theory to match, and the flexibility of allowing a third, “sub-tropical” matching layer. While writing up this result, I discovered a gap in the matching theory which I have not yet been able to surmount. In collaboration with R. Klein (Free University Berlin), one of the originators of the multi scale theories of tropical dynamics, we have been working to bridge the gap in the matching theory.

The new equations describe Rossby wave packets confined to the upper or lower troposphere for which a KdV soliton is an exact solution. In [20] I numerically investigated the interaction of Rossby solitons in the presence of constant wind shear either in the upper or lower troposphere, or interacting with one another. The most interesting and novel behavior demonstrated by these solutions are oscillations between upper and lower troposphere solitons in the presence of a mean wind shear.

During the summer of 2018, I mentored 2 students, Polina Rychenkova and Ruicong (Claire) Zheng, in a project exploring the Riemann ellipsoids - which are the classical self-gravitating ellipsoidal solutions to the equations of fluid dynamics.

(2) Nonlinear PDEs
Nonlinear Surface Waves on Vorticity Discontinuities

Two dimensional fluid dynamics is described by a nonlinear, non-local partial differential equation for the vorticity of the flow. If the flow consists of vortex patches, within which the vorticity is constant, then the dynamics can be reduced to equations on the interfaces of those patches. A circular vortex with constant vorticity or a straight line vorticity discontinuity in the plane behave similarly when perturbed a small amount - they admit waves whose frequency is independent of their wavenumber. The properties of these nondispersive waves differ from hyperbolic nondispersive waves. In collaboration with John Hunter at UC Davis, we developed a description of the nonlinear evolution of such small perturbations [21]. Using multiple scale asymptotics, we derived nonlinear, nonlocal asymptotic equation governing small perturbations of the interface and showed how this equation also arises in the weakly nonlinear asymptotics or through a near identity transformation of the Burgers-Hilbert equation so that the latter provides an effective equation for the vorticity discontinuity problem. Our numerical simulations of the Burgers-Hilbert and the asymptotic equation show that they are in excellent agreement and, in particular, they both develop a singularity at corresponding times.

We are in the process of investigating this singularity by performing numerical simulations on the equations describing the vortex interface using a Lagrangian numerical method which I developed. Since the interface is constantly flapping, it is not at all obvious what a singularity in the asymptotic equation, which describes the height of an interface which is single valued as a function of coordinate, would correspond to in the vorticity dynamics: we have found that the answer is filaments. The interface becomes multiple valued over this small range and, as it oscillates, an extremely thin filament is formed. At each successive extremum of the oscillation, another filament is shed. This filamentation cannot be described by the amplitude equation and since it occurs on such small scales, it requires an enormous amount of numerical adaptivity to carefully resolve.

We are extending this analysis to non-local, non-linear PDEs of lower regularity, such as the surface quasi-geostrophic equations.

Laplace’s equation with many immersed spherical boundaries.

Along with R. Samson (emeritus Shell Research and Technology, Amsterdam), we developed a theory based on off-center monopoles for the steady-state diffusion equation and for the convection-diffusion equation with a constant flow field. This exceedingly simple approach had not been previously explored and in Biello & Samson (2015) we show that it is exceedingly accurate for arbitrary number of spheres in the low Peclet number regime.

We extended our work to high Peclet numbers in [27] in SIAM J. Applied Mathematics. To extend our solution strategy to large Peclet number required careful matched asymptotic expansions for the advection/diffusion equation around a sphere. The near/far field matching allowed us to create a universal function of the concentration due to one spherical absorber downstream of itself in terms of the (Peclet) rescaled downstream coordinate. This allows a single pre-processing step (or table lookup) to compute the downstream concentration to allow the computation of the concentration in the presence of multiple absorbers.

We used similar asymptotic methods to study a the instability of multicomponent flow down a pipe in reference [29].
(3) **Service**

Department service includes FRC (chair of FRC from 2015-2016), GGAM executive committee as well as qualifying exam committees. I have been a very active undergraduate advisor for about 10 years now and have served as Head Adviser from Fall 2017.

During the last 3 years, my service to the Academic Senate has been significant and has consumed a significant amount of time. I am very proud to say that this time was well spent and I will explain below.

For the last two years, and also in years past, I have served as the chair of the UC Davis Preparatory Education Committee (PEC) of the Academic Senate. This committee is a permanent subcommittee of Undergraduate Council (UGC), and as chair of PEC I was a member of UGC. Furthermore, each of the individual campus chairs of PEC serve on the systemwide Committee on Preparatory Education in Oakland (UCOPE).

I have dedicated my last two years on these committees to pursuing reform of the evaluation and delivery of the Entry Level Writing Requirement at UC Davis. The arcana of the writing requirement do not fit into the context of a candidate’s statement. Suffice it to say that the requirement was being poorly delivered (as evidenced by data), was frustrating the students, was not providing good long term writing outcomes for the students, and, by virtue of the fact that it was being delivered outside of UC Davis, had avoided Senate oversight for over 11 years.

The reform of this program required a significant effort in understanding the details of Davis and Systemwide Senate regulations, convincing stakeholders of the need to make changes, convincing the administration that it could be cost effective, writing the necessary regulation and bylaws to implement the changes, and shepherding these changes through the Academic Senate. I am proud to say that I, along with my many partners in departments across L&S, was successful in this endeavor and I am firmly convinced that the new framework of writing instruction will provide a modern, accountable writing pedagogy for all of our students. I personally shepherded 2 regulation and 1 bylaw change through the UC Davis Academic Senate in order to allow the implementation of these reforms. I was also instrumental in forcing UCOPE to carefully consider the efficacy of the Analytic Writing Placement Exam which is used as the benchmark to place all UC students in writing instruction.

(3) **Diversity Statement**

Regarding diversity I am currently working with 2 female graduate students, Xiaoyun Niu and Jennifer Piaseczny. During the review period I graduated one female graduate student, Amanda Back. The postdoc with whom I’m collaborating, Jorge Guerra, is from an under-represented minority group. This past summer I mentored 2 women in undergraduate research, Ruicong Zheng and Polina Khapikova. The writing reform I pursued for the academic senate was intended to help fix a massive inequity for underprivileged first year students that resulted from the existing Entry Level Writing Program.

(3) **Teaching**

I have taught mostly Vector Calculus, Partial Differential Equations, and Ordinary Differential Equations over the last three years. I have worked hard to change the philosophy of lower division mathematics from a “plug and chug” approach (with which many students feel comfortable) to one
of demanding understanding and connections to real world problem. Most of our students are physical scientists or engineers and connecting math to the world for them is essential for their future education. My students have responded to this in a typical bi-modal fashion - some dislike my approach, but a core group feel absolutely inspired. I quote from a letter I received from one of my students,

“Hello Professor!
I just wanted to give you a big thanks for this interesting quarter. Even though it was hard getting up every morning to go to your lectures, your loud voice and enthusiasm made it worth it. .... Your lecture style for math is very different from anything from I have ever experienced (even though I’ve only been in Davis for 2 quarters hah!) but your focus on helping students understand principles is extremely commendable to me. Your enthusiasm for the course and willingness to help felt genuine. I remember one morning where I nonchalantly greeted you as we walked into Giedt and you replied with ten-fold of my energy! I just knew that day was going to be a great day.

I hope to elicit this kind of response in all of my students."